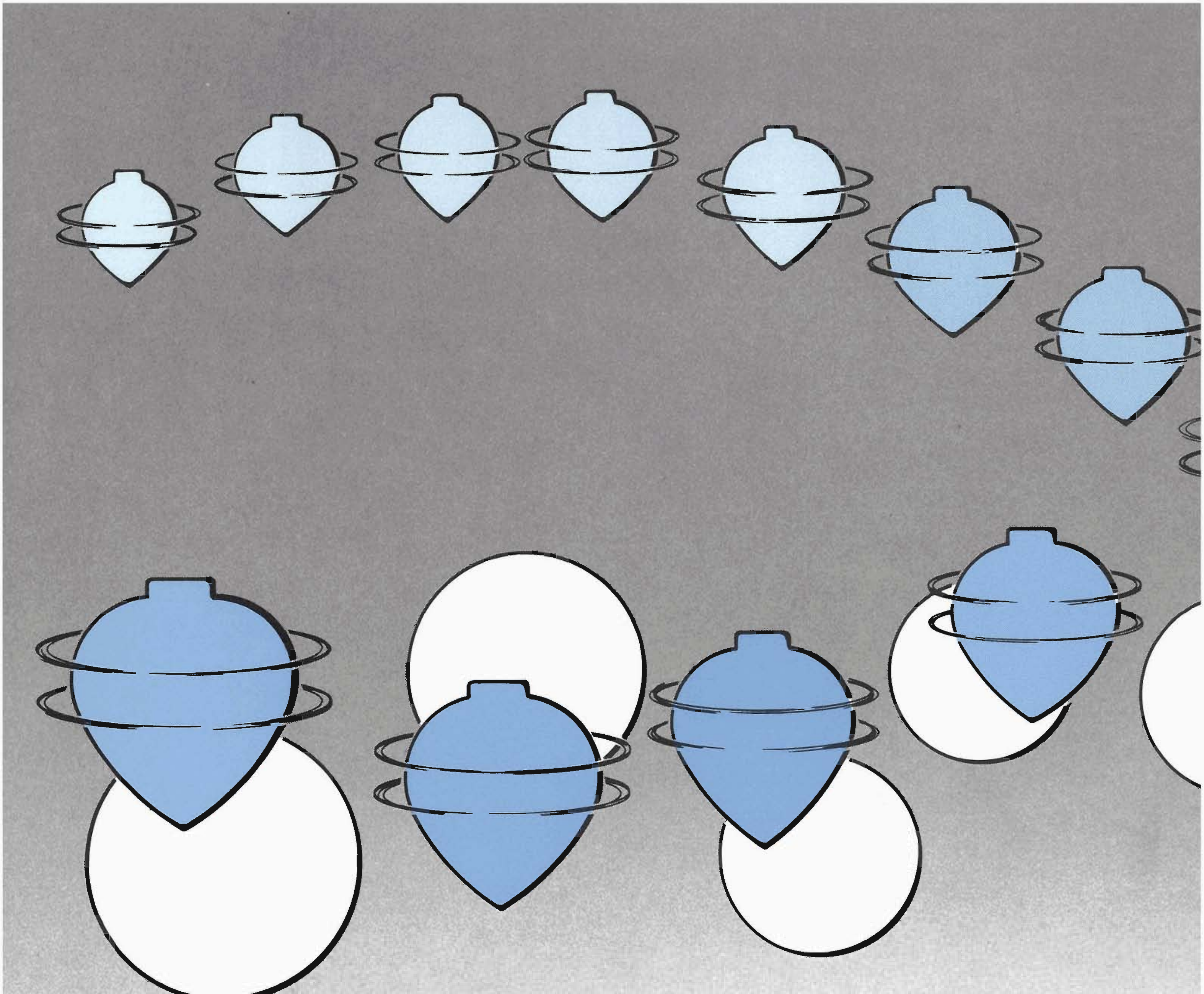
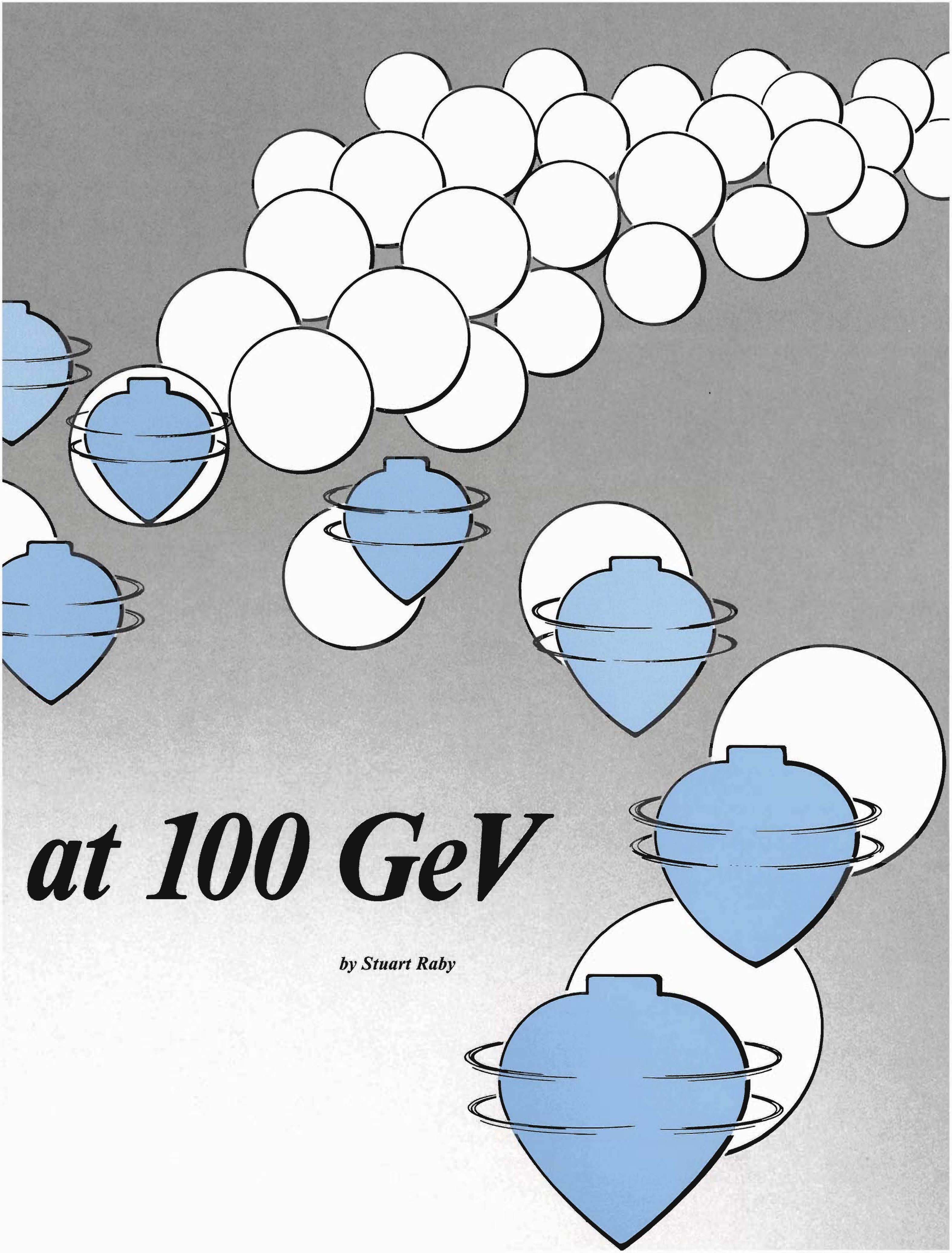


Supersymmetry

Supersymmetry is a symmetry that connects particles of integral and half-integral spin. Invented about ten years ago by physicists in Europe and the Soviet Union, supersymmetry was immediately recognized as having amazing dynamical properties. In particular, this symmetry provides a rational framework for unifying *all* the known forces between elementary particles—the strong, weak, electromagnetic, and gravitational. Indeed, it may also unify the separate concepts of matter and force into one comprehensive framework.

In the supersymmetric world depicted here, each boson pairs with a fermion partner.



at 100 GeV

by Stuart Raby

There are two types of symmetries in nature: external (or space-time) symmetries and internal symmetries. Examples of internal symmetries are the symmetry of isotopic spin that identifies related energy levels of the nucleons (protons and neutrons) and the more encompassing SU(3) × SU(2) × U(1) symmetry of the standard model (see "Particle Physics and the Standard Model"). Operations with these symmetries do not change the space-time properties of a particle.

External symmetries include translation invariance and invariance under the Lorentz transformations. Lorentz transformations, in turn, include rotations as well as the *special* Lorentz transformations, that is, a "boost" or a change in the velocity of the frame of reference.

Each symmetry defines a particular operation that does not affect the result of any experiment. An example of a spatial translation is to, say, move our laboratory (accelerators and all) from Chicago to New Mexico. We are, of course, not surprised that the result of any experiment is unaffected by the move, and we say that our system is translationally invariant. Rotational invariance is similarly defined with respect to rotating our apparatus about any axis. Invariance under a special Lorentz transformation corresponds to finding our results unchanged when our laboratory, at rest in our reference frame, is replaced by one moving at a constant velocity.

Corresponding to each symmetry operation is a quantity that is conserved. Energy and momentum are conserved because of time and space-translational invariance, respectively. The energy of a particle at rest is its mass ($E = mc^2$). Mass is thus an intrinsic property of a particle that is conserved because of invariance of our system under space-time translations.

Spin. Angular momentum conservation is a result of Lorentz invariance (both rotational and special). Orbital angular momentum refers to the angular momentum of a particle in motion, whereas the intrinsic angular momentum of a particle (remaining even at rest) is called spin. (Particle spin is an external symmetry, whereas isotopic spin, which is not based on Lorentz invariance, is not.)

In quantum mechanics spin comes in integral or half-integral multiples of a fundamental unit $\hbar$ ($\hbar = h/2\pi$ where h is Planck's constant). (Orbital angular momentum only comes in integral multiples of $\hbar$.) Particles with integral values of spin ($0, \hbar, 2\hbar, \ldots$) are called bosons, and those with half-integral spins ($\hbar/2, 3\hbar/2, 5\hbar/2, \ldots$) are called fermions. Photons (spin 1), gravitons (spin 2), and pions (spin 0) are examples of bosons. Electrons, neutrinos, quarks, protons, and neutrons—the particles that make up ordinary matter—are all spin-½ fermions.

The conservation laws, such as those of energy, momentum, or angular momentum, are very useful concepts in physics. The following example dealing with spin and the conservation of angular momentum provides one small bit of insight into their utility.

In the process of beta decay, a neutron decays into a proton, an electron, and an antineutrino. The antineutrino is massless (or very close to being massless), has no charge, and interacts only very weakly with other particles. In short, it is practically invisible, and for many years beta decay was thought to be simply

$$n \rightarrow p + e^- .$$

However, angular momentum is not conserved in this process since it is not possible for the initial angular momentum (spin 1/2 for the neutron) to equal the final total angular momentum (spin 1/2 for the proton ± spin 1/2 for the electron ± an integral value for the orbital angular momentum). As a result, W. Pauli predicted that the neutrino must exist because its half-integral spin restores conservation of angular momentum to beta decay.

There is a dramatic difference between the behavior of the two groups of spin-classified particles, the bosons and the fermions. This difference is clarified in the so-called spin-statistics theorem that states that bosons must satisfy commutation relations (the quantum mechanical wave function is symmetric under the interchange of identical bosons) and that fermions must satisfy anti-commutation relations (antisymmetric wave functions). The ramification of this simple statement is that an indefinite number of bosons can exist in the same place at the same time, whereas only one fermion can be in any given place at a given time (Fig. 1). Hence "matter" (for example, atoms) is made of fermions. Clearly, if you can't put more than one in any given place at a time, then they must take up space. If they are also observable in some way, then this is exactly our concept of matter. Bosons, on the other hand, are associated with "forces." For example, a large number of photons in the same place form a macroscopically observable electromagnetic field that affects charged particles.

Supersymmetry. The fundamental property of supersymmetry is that it is a space-time symmetry. A supersymmetry operation alters particle spin in half-integral jumps, changing bosons into fermions and vice versa. Thus supersymmetry is the first symmetry that can unify matter and force, the basic attributes of nature.

If supersymmetry is an exact symmetry in nature, then for every boson of a given mass there exists a fermion of the same mass and vice versa; for example, for the electron there should be a *scalar* electron (selectron), for the neutrino, a *scalar* neutrino (sneutrino), for quarks, *scalar* quarks (squarks), and so forth. Since no such degeneracies have been observed, supersymmetry cannot be an exact symmetry of nature. However, it might be a symmetry that is inexact or broken. If so, it can be broken in either of two inequivalent ways: *explicit* supersymmetry breaking in which the Lagrangian contains explicit terms that are not supersymmetric, or *spontaneous* supersymmetry breaking in which the Lagrangian is supersymmetric but the vacuum is not (spontaneous symmetry breaking is

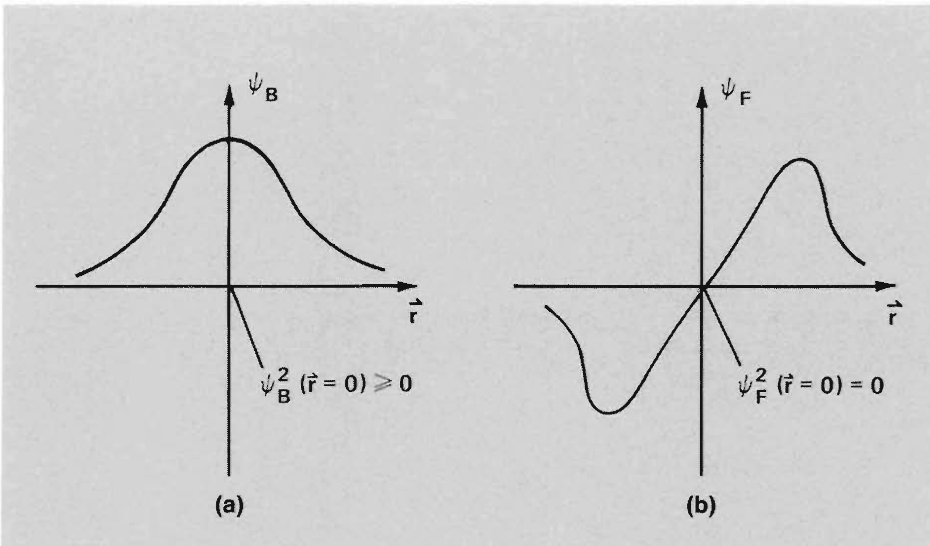


Fig. 1. (a) An example of a symmetric wave function for a pair of bosons and (b) an antisymmetric wave function for a pair of fermions, where the vector* r *represents the distance between each pair of identical particles. Because the boson wave function is symmetric with respect to exchange (ψ_B (r) = ψ_B(−r)), there can be a nonzero probablity (ψ^2_B) for two bosons to occupy the same position in space (r = 0), whereas for the asymmetric fermion wave function (ψ_F (r) = −ψ_F (−r)) the probability (ψ^2_F) of two fermions occupying the same position in space* must *be zero.

explained in Notes 3 and 6 of "Lecture Notes—From Simple Field Theories to the Standard Model"). Either way will lift the boson-fermion degeneracy, but the latter way will introduce (in a somewhat analogous way to the Higgs boson of weak-interaction symmetry breaking) a new particle, the Goldstone fermion. (We develop mathematically some of the ideas of this paragraph in "Supersymmetry and Quantum Mechanics".)

A question of extreme importance is the scale of supersymmetry breaking. This scale can be characterized in terms of the so-called *supergap*, the mass splitting between fermions and their bosonic partners ($\delta^2 = M^2_{\mathrm{B}} - M^2_{\mathrm{F}}$). Does one expect this scale to be of the order of the weak scale (~100 GeV), or is it much larger? We will discuss the first possiblity at length because if supersymmetry is broken on a scale of order 100 GeV there are many predictions that can be verified in the next generation of high-energy accelerators. The second possibility would not necessarily lead to any new low-energy consequences.

We will also discuss the role gravity has played in the description of low-energy supersymmetry. This connection betweeen physics at the largest mass scale in nature (the Planck scale: $M_{\mathrm{pl}} = (\hbar c/G_{\mathrm{N}})^{1/2} \simeq 1.2 \times 10^{19}$ GeV/c^2, where G_{N} is Newton's gravitational constant) and physics at the low energies of the weak scale ($M_W \simeq 83$ GeV/c^2 where M_W is the mass of the W boson responsible for weak interactions) is both novel and exciting.

Motivations. Why would one consider supersymmetry to start with?

First, supersymmetry is the largest possible symmetry of nature that can combine internal symmetries and space-time symmetries in a nontrivial way. This combination is *not* a necessary feature of supersymmetry (in fact, it is accomplished by extending the algebra of Eqs. 2 and 3 in "Supersymmetry and Quantum Mechanics" to include more supersymmetry generators and internal symmetry generators). However, an important consequence of such an extension might be that bosons and fermions in different representations of an internal symmetry group are related. For example, quarks (fermions) are in triplets in the strong-interaction group SU(3), whereas the gluons (bosons) are in octets. Perhaps they are all related in an extended supersymmetry, thus providing a unified description of quarks and their forces.

Second, supersymmetry can provide a theory of gravity. If supersymmetry is global, then a given supersymmetry rotation must be the same over all space-time. However, if supersymmetry is local, the system is invariant under a supersymmetry rotation that may be arbitrarily different at every point. Because the various generators (supersymmetry charges, four-momentum translational generators, and Lorentz generators for both rotations and boosts) satisfy a common algebra of commutation and anticommutation relations, consistency *requires* that all the symmetries are local. (In fact, the anticommutator of two supersymmetry generators is a translation generator.) Thus different points in space-time can transform in different ways; put simply, this can amount to acceleration between points, which, in turn, is equivalent to gravity. In fact, the theory of local translations and Lorentz transformations is just general relativity, that is, Einstein's theory of gravity, and a supersymmetric theory of gravity is called supergravity. It is just the theory invariant under local supersymmetry. Thus, supersymmetry allows for a possible unification of all of nature's particles and their interactions.

These two motivations were realized quite soon after the advent of supersymmetry. They are possibilities that unfortunately have not yet led to any reasonable predic-

continued on page 106

Supersymmetry in Quantum Mechanics

I intend to develop here some of the algebra pertinent to the basic concepts of supersymmetry. I will do this by showing an analogy between the quantum-mechanical harmonic oscillator and a bosonic field and a further analogy between the quantum-mechanical spin-½ particle and a fermionic field. One result of combining the two resulting fields will be to show that a "tower" of degeneracies between the states for bosons and fermions is a natural feature of even the simplest of supersymmetry theories.

A supersymmetry operation changes bosons into fermions and vice versa, which can be represented schematically with the operators $Q_\alpha^\dagger$ and Q_α and the equations

$$Q_\alpha^\dagger|\text{boson}\rangle = |\text{fermion}\rangle_\alpha$$

$$\text{and} \qquad (1)$$

$$Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle_\alpha \, .$$

In the simplest version of supersymmetry, there are four such operators or generators of supersymmetry (Q_α and the Hermitian conjugate $Q_\alpha^\dagger$ with $\alpha = 1, 2$). Mathematically, the generators are Lorentz spinors satisfying fermionic anticommutation relations

$$\{Q_\alpha^\dagger, Q_\beta\} = p^\mu \, (\sigma_\mu)_{\alpha\beta} \, , \qquad (2)$$

where p^μ is the energy-momentum four-vector ($p^0 = H$, p^i = three-momentum) and the σ_μ are two-by-two matrices that include the Pauli spin matrices σ^i ($\sigma_\mu = (1, \sigma^i)$ where $i = 1, 2, 3$). Equation 2 represents the unusual feature of this symmetry: the supersymmetry operators combine to generate translation in space and time. For example, the operation of changing a fermion to a boson and back again results in changing the position of the fermion.

If supersymmetry is an invariance of nature, then

$$[H, Q_\alpha] = 0 \, , \qquad (3)$$

that is, Q_α commutes with the Hamiltonian H of the universe. Also, in this case, the vacuum is a supersymmetric singlet ($Q_\alpha|\text{vac}\rangle = 0$).

Equations 1 through 3 are the basic defining equations of supersymmetry. In the form given, however, the supersymmetry is solely an external or space-time symmetry (a supersymmetry operation changes particle spin without altering any of the particle's internal symmetries). An extended supersymmetry that connects external and internal symmetries can be constructed by expanding the number of operators of Eq. 2. However, for our purposes, we need not consider that complication.

The Harmonic Oscillator. In order to illustrate the consequences of Eqs. 1 through 3, we first need to review the quantum-mechanical treatment of the harmonic oscillator.

The Hamiltonian for this system is

$$H_{\text{osc}} = \frac{1}{2}\,(p^2 + \omega^2 q^2) \, , \qquad (4)$$

where p and q are, respectively, the momentum and position coordinates of a nonrelativistic particle with unit mass and a $2\pi/\omega$ period of oscillation. The coordinates satisfy the quantum-mechanical commutation relation

$$[p, q] = (pq - qp) = -i\hbar \; . \tag{5}$$

The well-known solution to the harmonic oscillator (the set of eigenstates and eigenvalues of H_{osc}) is most conveniently expressed in terms of the so-called raising and lowering operators, $a^\dagger$ and a, respectively, which are defined as

$$a^\dagger = \frac{1}{\sqrt{2\omega\hbar}}\,(p + i\omega q)$$

and (6)

$$a = \frac{1}{\sqrt{2\omega\hbar}}\,(p - i\omega q) \, ,$$

and which satisfy the commutation relation

$$[a, a^\dagger] = 1 \; . \tag{7}$$

In terms of these operators, the Hamiltonian becomes

$$H_{osc} = \hbar\omega(a^\dagger a + ½) \, , \tag{8}$$

with eigenstates

$$|n\rangle = N_n(a^\dagger)^n|0\rangle \, , \tag{9}$$

where N_n is a normalization factor and $|0\rangle$ is the ground state satisfying

$$a|0\rangle = 0$$

and (10)

$$\langle 0|0\rangle = 1 \; .$$

It is easy to show that

$$a^\dagger|n\rangle = \sqrt{n+1}\;|n+1\rangle$$

and (11)

$$a|n\rangle = \sqrt{n}\;\;|n-1\rangle \, ,$$

hence the names raising operator for $a^\dagger$ and lowering operator for a. Also note that $a^\dagger a$ is just a counting operator since $a^\dagger\, a\,|n\rangle = n\,|\,n\rangle$. Finally, we find that

$$H_{osc}\,|n\rangle = \hbar\omega(n + ½)\,|n\rangle \, , \tag{12}$$

that is, the states $|n\rangle$ have energy $(n + ½)\,\hbar\omega$.

The Bosonic Field. There is a simple analogy between the quantum oscillator and the scalar quantum field needed to represent bosons (scalar particles). A free scalar field is quite rigorously described by an infinite set of noninteracting harmonic oscillators $\{a_p^\dagger, a_p\}$, where p is an index labeling the set. The Hamiltonian of the free field can be written as

$$H_{scalar} = \sum_p \hbar\omega_p\left(a_p^\dagger\, a_p + ½\right) , \tag{13}$$

with the summation taken over the individual oscillators p.

The ground state of the free scalar quantum field is called the vacuum (it contains no scalar particles) and is described mathematically by the conditions

$$a_p\,|\mathrm{vac}\rangle = 0$$

and (14)

$$\langle\mathrm{vac}|\mathrm{vac}\rangle = 1 \; .$$

The $a_p^\dagger$ and a_p operators create or annihilate, respectively, a single scalar particle with energy $\hbar\omega_p$ ($\hbar\omega_p = \sqrt{p^2+m^2}$, where p is the momentum carried by the created particle and m is the mass). A scalar particle is thus an excitation of one particular oscillator mode.

The Fermionic Field. The simple quantum-mechanical analogue of a spin-½ field needed to represent fermions is just a quantum particle with spin ½. This is necessary because, whereas bosons can be represented by scalar particles satisfying commutation relations, fermions must be represented by spin-½ particles satisfying anticommutation relations.

A spin-½ particle has two spin states: $|0\rangle$ for spin down and $|1\rangle$ for spin up. Once again we define raising and lowering operators, here $b^\dagger$ and b, respectively. These operators satisfy the anticommutation relations

$$\{b, b^\dagger\} = (bb^\dagger + b^\dagger b) = 1$$

and (15)

$$\{b^\dagger, b^\dagger\} = \{b, b\} = 0 \,.$$

If $b|0\rangle = 0$, it is easy to show that

$$b^\dagger\,|0\rangle = |1\rangle$$

and (16)

$$b^\dagger\,|1\rangle = |0\rangle \,,$$

where $b^\dagger\, b$ is again a counting operator satisfying

$$b^\dagger b\,|1\rangle = |1\rangle$$

and (17)

$$b^\dagger b\,|0\rangle = 0 \,.$$

We may define a Hamiltonian

$$H_{\text{spin}} = \hbar\omega(b^\dagger b - \tfrac{1}{2}) \,, \tag{18}$$

so that states $|1\rangle$ and $|0\rangle$ will have energy equal to $\frac{1}{2}\hbar\omega$ and $-\frac{1}{2}\hbar\omega$, respectively.

The analogy between the free quantum-mechanical fermionic field and the simple quantum-mechanical spin-½ particle is identical to the scalar field case. For example, once again we may define an infinite set $\{b_p^\dagger, b_p\}$ of noninteracting spin-½ particles labeled by the index p. The vacuum state satisfies

$$b_p\,|\text{vac}\rangle = 0$$

and (19)

$$\langle\text{vac}|\text{vac}\rangle = 1 \,.$$

Here $b_p^\dagger$ and b_p are identified as creation and annihilation operators, respectively, of a single fermionic particle. Note that since $\{b_p^\dagger, b_p^\dagger\} = 0$, it is only possible to create *one* fermionic particle in the state p. This is the Pauli exclusion principle.

Supersymmetry. Let us now construct a simple supersymmetric quantum-mechanical system that includes the bosonic oscillator degrees of freedom ($a^\dagger$ and a) and the fermionic spin-½ degrees of freedom ($b^\dagger$ and b). We define the anticommuting charges

$$Q = a^\dagger b(\hbar\omega)^{1/2}$$

and (20)

$$Q^\dagger = ab^\dagger(\hbar\omega)^{1/2} \,.$$

It is then easy to verify that

$$\begin{aligned}\{Q^\dagger, Q\} &= H = H_{\text{osc}} + H_{\text{spin}} \\ &= \hbar\omega(a^\dagger a + b^\dagger b) \,,\end{aligned} \tag{21}$$

and

$$[H, Q] = 0 \,. \tag{22}$$

Equations 21 and 22 are the direct analogues of Eqs. 2 and 3, respectively. We see that the anticommuting charges Q combine to form the generator of time translation, namely, the Hamiltonian H. The ground state of this system is the state $|0\rangle_{\text{osc}}|0\rangle_{\text{spin}} = |0,0\rangle$, where both the oscillator and the spin-½ degrees of freedom are in the lowest energy state. This state is a unique one, satisfying

$$Q|0,0\rangle = Q^\dagger|0,0\rangle = 0 \,. \tag{23}$$

The excited states form a tower of degenerate levels (see figure) with energy $(n + \frac{1}{2})\hbar\omega \pm \frac{1}{2}\hbar\omega$, where the sign of the second term is determined by whether the spin-½ state is $|1\rangle$ (plus) or $|0\rangle$ (minus).

The tower of states illustrates the boson-fermion degeneracy for exact supersymmetry. The bosonic states $|n{+}1,0\rangle$ (called bosonic in the field theory analogy because they contain no fermions) have the same energy as their fermionic partners $|n,1\rangle$.

Moreover, it is easy to see that the charges Q and $Q^\dagger$ satisfy the relations

$$Q|n,1\rangle = \sqrt{n+1}\,|n+1,0\rangle$$

and

Energy	States	
	Boson	Fermion
0	$\|0,0\rangle$	
$\hbar\omega$	$\|1,0\rangle$	$\|0,1\rangle$
$2\hbar\omega$	$\|2,0\rangle$	$\|1,1\rangle$
$3\hbar\omega$	$\|3,0\rangle$	$\|2,1\rangle$
.	.	.
.	.	.
.	.	.

The boson-fermion degeneracy for exact supersymmetry in which the first number in $|n,m\rangle$ corresponds to the state for the oscillator degree of freedom (the scalar, or bosonic, field) and the second number to that for the spin-½ degree of freedom (the fermionic field).

$$Q^\dagger|n+1,0\rangle = \sqrt{n+1}\;|n,1\rangle\,, \tag{24}$$

which are analogous to Eq. 1 because they represent the conversion of a fermionic state to a bosonic state and vice versa.

The above example is a simple representation of supersymmetry in quantum mechanics. It is, however, trivial since it describes non-interacting bosons (oscillators) and fermions (spin-½ particles). Non-trivial *interacting* represntations of supersymmetry may also be obtained. In some of these representations it is possible to show that the ground state is not supersymmetric even though the Hamiltonian is. This is an example of spontaneous supersymmetry breaking.

Symmetry Breaking. If supersymmetry were an exact symmetry of nature, then bosons and fermions would come in degenerate pairs. Since this is not the case, the symmetry must be broken. There are two inequivalent ways in which to do this and thus to have the degeneracy removed.

First we may add a small symmetry breaking term to the Hamiltonian, that is, $H \rightarrow H + \varepsilon H'$, where ε is a small parameter and

$$[H', Q] \neq 0\,. \tag{25}$$

This mechanism is called *explicit symmetry breaking.* Using it we can give scalars a mass that is larger than that of their fermionic partners, as is observed in nature. Although this breaking mechanism may be perfectly self-consistent (even this is in doubt when one includes gravity), it is totally ad hoc and lacks predictive power.

The second symmetry breaking mechanism is termed *spontaneous symmetry breaking.* This mechanism is characterized by the fact that the Hamiltonian remains supersymmetric,

$$[Q,H] = 0\,, \tag{26}$$

but the ground state does not,

$$Q|\mathrm{vac}\rangle \neq 0\,. \tag{27}$$

Supersymmetry can either be a global symmetry, such as the rotational invariance of a ferromagnet, or a local symmetry, such as a phase rotation in electrodynamics. Spontaneous breaking of a *global* symmetry leads to a massless Nambu-Goldstone particle. In supersymmetry we obtain a massless fermion $\tilde{G}$, the goldstino.

Spontaneous breaking of a *local* symmetry, however, results in the gauge particle becoming massive. (In the standard model, the W bosons obtain a mass $M_W = gV$ by "eating" the massless Higgs bosons, where g is the SU(2) coupling constant and V is the vacuum expectation value of the neutral Higgs boson.) The gauge particle of local supersymmetry is called a gravitino. It is the spin-3/2 partner of the graviton; that is, local supersymmetry incorporates Einstein's theory of gravity. When supersymmetry is spontaneously broken, the gravitino obtains a mass

$$m_G = G_N^{1/2}\Lambda_{ss}^2 \tag{28}$$

by "eating" the goldstino (here G_N is Newton's gravitational constant and Λ_{ss} is the vacuum expectation of some field that spontaneously breaks supersymmetry).

Thus, if the ideas of supersymmetry are correct, there is an underlying symmetry connecting bosons and fermions that is "hidden" in nature by spontaneous symmetry breaking. ■

continued from page 101

tions. Many workers in the field are, however, still pursuing these elegant notions.

Recently a third motivation for supersymmetry has been suggested. I shall describe the motivation and then discuss its expected consequences.

For many years Dirac focused attention on the "problem of large numbers" or, more recently, the "hierarchy problem." There are many extremely large numbers that appear in physics and for which we currently have no good understanding of their origin. One such large number is the ratio of the gravitational and weak-interaction mass scales mentioned earlier ($M_{pl}/M_W \sim 10^{17}$).

The gravitational force between two particles is proportional to the product of the energy (or mass if the particles are at rest) of the two particles times G_N. Thus, since $G_N \propto 1/M_{pl}^2$, the force between two W bosons at rest is proportional to $M_W^2/M_{pl}^2 \sim 10^{-34}$. This is to be compared to the electric force between W bosons, which is proportional to $\alpha = e^2/(4\pi\hbar c) \sim 10^{-2}$, where e is the electromagnetic coupling constant. Hence gravitational interactions between all known elementary particles are, at observable energies, at least 10^{32} times weaker than their electromagnetic interactions.

The key word is observable, for if we could imagine reaching an energy of order $M_{pl}c^2$, then the gravitational interactions would become quite strong. In other words, gravitationally bound states can be formed, in principle, with mass of order $M_{pl} \sim 10^{19}$ GeV. The Planck scale might thus be associated with particles, as yet unobserved, that have strong gravitational interactions.

At a somewhat lower energy, we also have the grand unification scale ($M_G \sim 10^{15}$ GeV or greater), another very large scale with similar theoretical significance. New particles and interactions are expected to become important at M_G.

In either case, should these new phenomena exist, we are faced with the question of why there are two such diverse scales, M_W and M_{pl} (or M_G), in nature.

The problem is exacerbated in the context of the standard model. In this mathematical framework, the W boson has a nonzero mass M_W because of spontaneous symmetry breaking and the existence of the scalar particle called the Higgs boson. Moreover, the mass of the W and the mass of the Higgs particle must be approximately equal. Unfortunately scalar masses are typically extremely sensitive to the details of the theory at very high energies. In particular, when one calculates quantum mechanical corrections to the Higgs mass μ_H in perturbation theory, one finds

$$\mu_H^2 = (\mu_H^0)^2 + \delta\mu^2 , \tag{1}$$

where

$$\delta\mu^2 \sim \alpha\, M_{\rm large}^2 . \tag{2}$$

In these equations μ_H^0 is the zeroth order value of the Higgs boson mass, which can be zero, and $\delta\mu^2$ is the perturbative correction. The parameter α is a generic coupling constant connecting the low mass states of order M_W and the heavy states of order $M_{\rm large}$, that is, the largest mass scale in the theory. For example, some of the theorized particles with mass M_{pl} or M_G will have electric charge and interact with known particles. In this case, $\alpha = e^2/4\pi\hbar c$, a measure of the electromagnetic coupling. Clearly μ_H is naturally very large here and *not* approximately equal to the mass of the W.

Supersymmetry can ameliorate the problem because, in such theories, scalar particles are no longer sensitive to the details at high energies. As a result of miraculous cancellations, one finds

$$\delta\mu^2 \sim \alpha\, (\mu_H^0)^2 \ln (M_{\rm large}) . \tag{3}$$

This happens in the following way (Fig. 2).

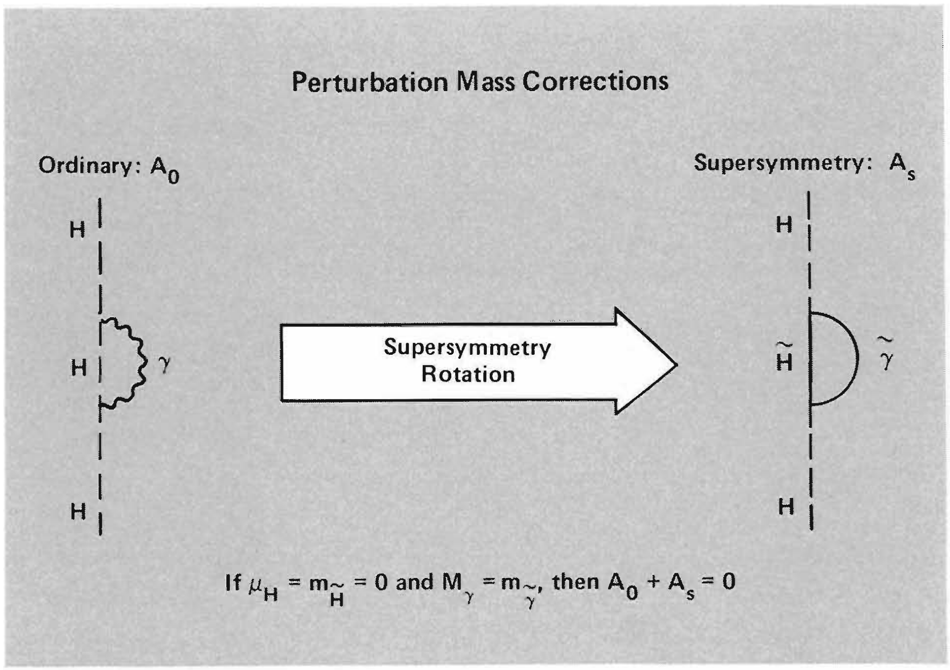


Fig. 2. If A_0 (left) represents a perturbative mass correction for an ordinary particle H due to the creation of a virtual photon γ, then a supersymmetry rotation of the central region of the diagram will generate a second mass correction A_s (right) involving the supersymmetric partners $\tilde{H}$ and the photino $\tilde{\gamma}$. If supersymmetry is an exact symmetry, then the total mass correction is zero.

For each ordinary mass correction, there will be a second mass correction related to the first by a supersymmetry rotation (the symmetry operation changes the virtual particles of the ordinary correction into their corresponding supersymmetric partners). Although each correction *separately* is proportional to $\alpha\, M^2_{\text{large}}$, the sum of the two corrections is given by Eq. 3. In this case, if $\mu^0_H = 0$, then $\mu_H = 0$ and will remain zero to all orders in perturbation theory as long as supersymmetry remains unbroken. Hence supersymmetry is a symmetry that prevents scalars from getting "large" masses, and one can even imagine a limit in which scalar masses vanish. Under these conditions we say scalars are "naturally" light.

How then do we obtain the spontaneous breaking of the weak interactions and a W boson mass? We remarked that supersymmetry cannot be an exact symmetry of nature; it must be broken. Once supersymmetry is broken, the perturbative correction (Eq. 3) is replaced by

$$\delta\mu^2 \sim \alpha\,(\mu^0_H)^2 \ln(M_{\text{large}}) + \alpha\,\Lambda^2_{\text{ss}}\,, \tag{4}$$

where Λ_{ss} is the scale of supersymmetry breaking. If supersymmetry is broken spontaneously, then Λ_{ss} is not sensitive to M_{large} and could thus have a value that is much less than M_{large}. This correction to the Higgs boson mass can then result in a spontaneous breaking of the weak interactions, with the standard mechanism, at a scale of order $\Lambda_{\text{ss}} \ll M_{\text{large}}$.

Table 1

The Supersymmetry Doubling of Particles

	Standard Model	**Supersymmetric Partners**	
spin-½ quarks	$\begin{pmatrix} u \\ d \end{pmatrix}$ $\bar{u}$ $\bar{d}$	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}$ $\tilde{\bar{u}}$ $\tilde{\bar{d}}$	spin-0 squarks
spin-½ leptons	$\begin{pmatrix} \nu \\ e \end{pmatrix}$ $\bar{e}$	$\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}$ $\tilde{\bar{e}}$	spin-0 sleptons
(There are two other quark-lepton families similar to this one.)			
spin-1 gauge bosons	γ, $W^{\pm}$, Z^0, g	$\tilde{\gamma}$, $\tilde{W}^{\pm}$, $\tilde{Z}^0$, $\tilde{g}$	spin-½ gauginos
spin-0 Higgs bosons	$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ $\begin{pmatrix} \bar{H}^- \\ \bar{H}^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}^+ \\ \tilde{H}^0 \end{pmatrix}$ $\begin{pmatrix} \tilde{\bar{H}}^- \\ \tilde{\bar{H}}^0 \end{pmatrix}$	spin-½ Higgsinos
	Global Supersymmetry		
spin-0 scalar partner	G	$\tilde{G}$ (massless)	spin-½ Goldstino
	Local Supersymmetry		
spin-0 scalar partner	G		
spin-2 graviton	g	$\tilde{G}$ (massive)	spin-3/2 gravitino

The Particles. We've discussed a bit of the motivation for supersymmetry. Now let's describe the consequences of the minimal supersymmetric extension of the standard model, that is, the particles, their masses, and their interactions.

The particle spectrum is literally doubled (Table 1). For every spin-½ quark or lepton there is a spin-0 scalar partner (squark or slepton) with the same quantum numbers under the $SU(3) \times SU(2) \times U(1)$ gauge interactions. (We show only the first family of quarks and leptons in Table 1; the other two families include the s, c, b, and t quarks, and, for leptons, the muon and tau and their associated neutrinos.)

The spin-1 gauge bosons (the photon γ, the weak interaction bosons $W^{\pm}$ and Z^0, and the gluons g) have spin-½ fermionic partners, called gauginos.

Likewise, the spin-0 Higgs boson, responsible for the spontaneous symmetry breaking of the weak interaction, should have a spin-½ fermionic partner, called a Higgsino. However, we have included two sets of weak *doublet* Higgs bosons, denoted H and $\bar{H}$, giving a total of four Higgs bosons and four Higgsinos. Although only one weak doublet of Higgs bosons is required for the weak breaking of the standard model, a consistent supersymmetry theory requires the two sets. As a result (unlike the standard model, which predicts one neutral Higgs boson), supersymmetry predicts that we should observe two *charged* and three neutral Higgs bosons.

Finally, other particles, related to symmetry breaking and to gravity, should be introduced. For a global supersymmetry, these particles will be a massless spin-½ Goldstino and its spin-0 partner. However, in the local supersymmetry theory needed for gravity, there will also be a graviton and its supersymmetric partner, the gravitino. We will discuss this point in greater detail later, but local symmetry breaking combines the Goldstino with the gravitino to form a massive, rather than a massless, gravitino.

In many cases the doubling of particles just outlined creates a supersymmetric partner that is absolutely stable. Such a particle

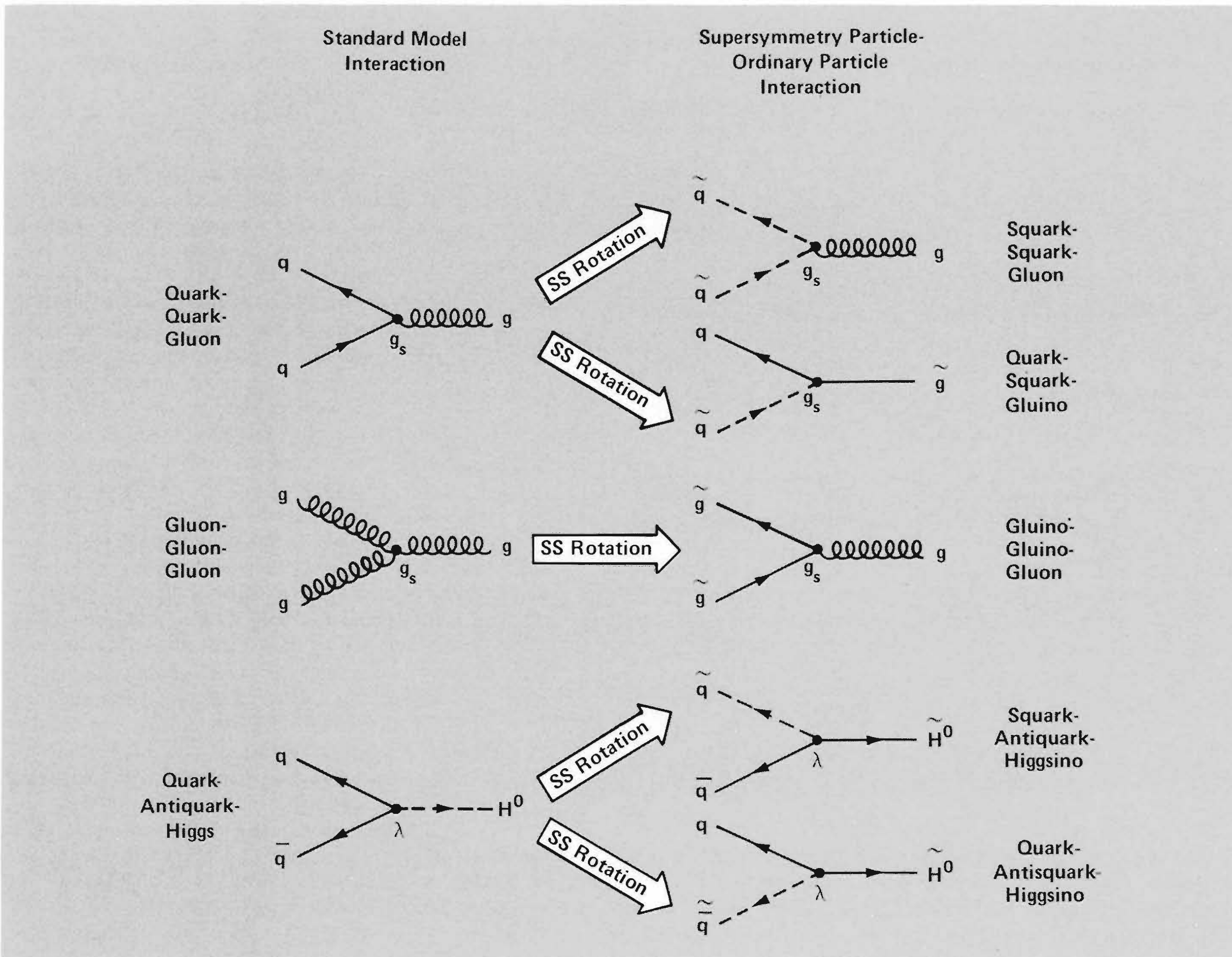


Fig. 3. Examples of interactions between ordinary particles (left) and the corresponding interactions between an ordinary particle and two supersymmetric particles (right) obtained by performing a supersymmetry rotation on the first interaction.

could, in fact, be the dominant form of matter in our universe.

The Masses. What is the expected mass for the supersymmetric partners of the ordinary particles? The theory, to date, does not make any firm predictions; we can nevertheless obtain an order-of-magnitude estimate in the following manner.

Although an unbroken supersymmetry can keep scalars massless, once supersymmetry is broken, all scalars obtain quantum corrections to their masses proportional to the supersymmetry breaking scale Λ_{ss}, that is

$$\delta\mu^2 \sim \alpha\, \Lambda_{ss}^2\,, \tag{5}$$

which is Eq. 4 with the first negligible term dropped. If we demand the Higgs mass $\mu_H^2 \sim \delta\mu^2$ to be of order M_W^2, then $\Lambda_{ss}^2 \sim M_W^2/\alpha$ is at most of order 1000 GeV. Moreover, the mass splitting between all ordinary particles and their supersymmetric partners is again of order M_W. We thus conclude that if supersymmetry is responsible for the large ratio

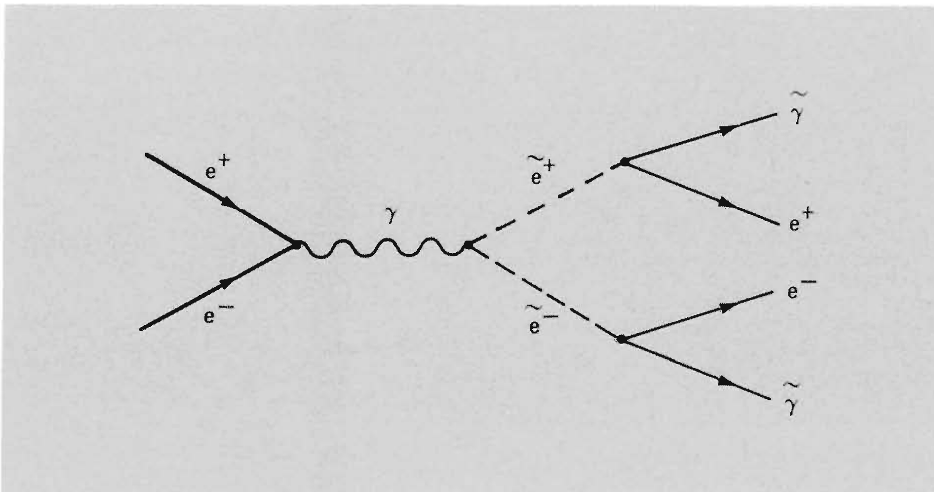


Fig. 4. A possible interaction involving supersymmetric particles (the selectrons $\tilde{e}^+$ and $\tilde{e}^-$ and the photino $\tilde{\gamma}$) that experimentally would be easily recognizable.

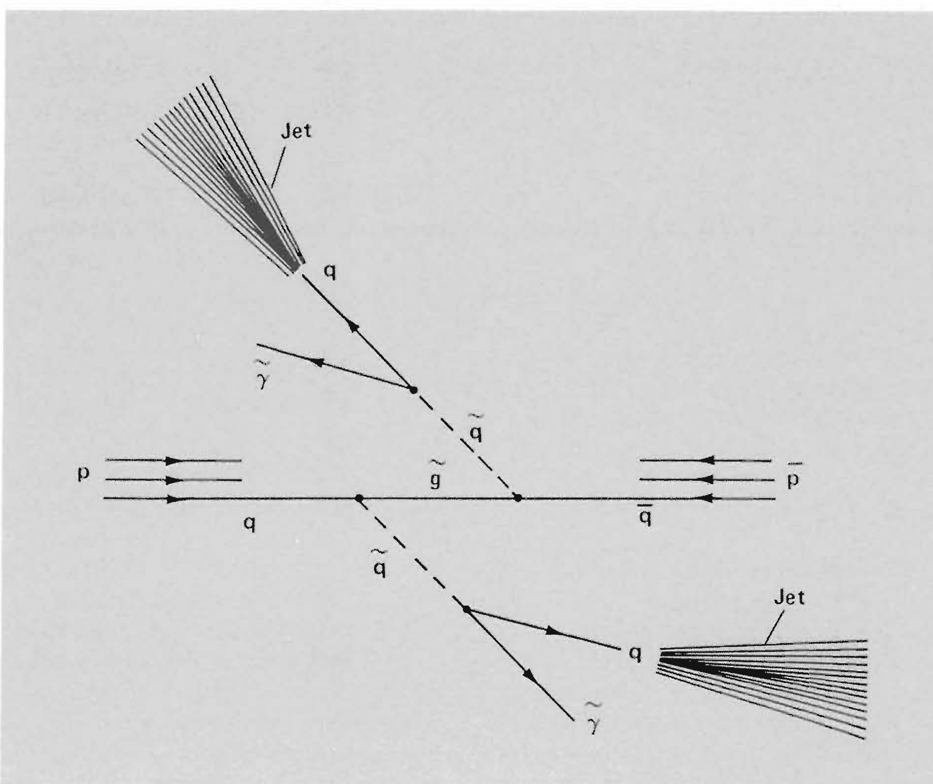


Fig. 5. A process involving supersymmetric particles (a gluino $\tilde{g}$ and squarks $\tilde{q}$) that generates two hadronic jets.

M_{pl}/M_W, then the new particles associated with supersymmetry will be seen in the next generation of high-energy accelerators.

The Interactions. As a result of supersymmetry, the entire low-energy spectrum of particles has been doubled, the masses of the new particles are of order M_W, but these masses cannot be predicted with any better accuracy. A reasonable person might therefore ask what properties, if any, *can* we predict. The answer is that we know all the interactions of the new particles with the ordinary ones, of which several examples are shown in Fig. 3. To get an interaction between ordinary and new particles, we can start with an interaction between three ordinary particles and rotate two of these (with a supersymmetry operation) into their supersymmetric partners. The important point is that as a result of supersymmetry the coupling constants remain unchanged.

Since we understand the interactions of the new particles with the ordinary ones, we know how to find these new objects. For example, an electron and a positron can annihilate and produce a pair of selectrons that subsequently decay into an electron-positron pair and two photinos (Fig. 4). This process is easily recognizable and would be a good signal of supersymmetry in high-energy electron-positron colliders.

Supersymmetry is also evident in the process illustrated in Fig. 5. Here one of the three quarks in a proton interacts with one of the quarks in an antiproton; the interaction is mediated by a gluino. The result is the generation of two squarks that decay into quarks and photinos. Because quarks do not exist as free particles, the experimenter should observe two hadronic jets (each jet is a collection of hadrons moving in the same direction as, and as a consequence of, the initial motion of a single quark). The two photinos will generally not interact in the detector, and thus some of the total energy of the process will be "missing".

The theories we have been discussing until now have been a minimal supersymmetric extension of the standard model. There are, however, two further extrapolations that are interesting both theoretically and phenomenologically. The first concerns gravity and the second, grand unified supersymmetry models.

Gravity. We have already remarked that supersymmetry may be either a global or a local symmetry. If it is a global symmetry, the Goldstino is massless and the lightest supersymmetric partner. However, if supersymmetry is a local symmetry, it necessarily includes the gravity of general relativity and the Goldstino becomes part of a massive gravitino (the spin-3/2 partner of the graviton) with mass

$$m_G \cong \frac{\Lambda_{ss}^2}{M_{pl}} . \tag{6}$$

With Λ_{ss} of order $M_W/\sqrt{\alpha}$ or 1000 GeV, m_G is extremely small ($\sim 10^{-10}$ times the mass of the electron).

Recently it was realized that under certain circumstances Λ_{ss} can be much larger than M_W, but, at the same time, the perturbative corrections $\delta\mu^2$ can still satisfy the constraint that they be of order M_W^2. In these special cases, supersymmetry breaking effects vanish in the limit as some very large mass diverges; that is, we obtain

$$\delta\mu^2 \sim \alpha\left(\frac{\Lambda_{ss}}{M_{large}}\right)^2 \tag{7}$$

instead of Eq. 5. An example is already provided by the gravitino mass m_G (where $M_{large} = M_{pl}$). In fact, models have now been constructed in which the gravitino mass is of order M_W and sets the scale of the low-energy supergap δ^2 between bosons and fermions.

In either case (an extremely small or a very large gravitino mass), the observation of a *massive* gravitino is a clear signal of local supersymmetry in nature, that is, the nontrivial extension of Einstein's gravity or supergravity.

Grand Unification. Our second extrapolation of supersymmetry has to do with grand unified theories, which provide a theoretically appealing unification of quarks and leptons and their strong, weak, and electromagnetic interactions. So far there has been one major experimental success for grand unification and two unconfirmed predictions.

The success has to do with the relationship between various coupling constants. In the minimal unification symmetry SU(5), two independent parameters (the coupling constant g_G and the value of the unification mass M_G) determine the three independent coupling constants (g_s, g, and g') of the standard-model SU(3) $\times$ SU(2) $\times$ U(1) symmetry. As a result, we obtain one prediction, which is typically expressed in terms of the weak-interaction parameter:

$$\sin^2\theta_W = \frac{g'^2}{g'^2 + g^2} . \tag{8}$$

The theory of minimal SU(5) predicts $\sin^2\theta_W = 0.21$, whereas the experimentally observed value is 0.22 ± 0.01, in excellent agreement.

The two predictions of SU(5) that have not been verified experimentally are the existence of magnetic monopoles and proton decay. The expected abundance of magnetic monopoles today is crucially dependent on poorly understood processes occurring in the first 10^{-35} second of the history of the universe. As a result, if they are not seen, we may ascribe the problem to our poor understanding of the early universe. On the other hand, if proton decay is not observed at the ex-

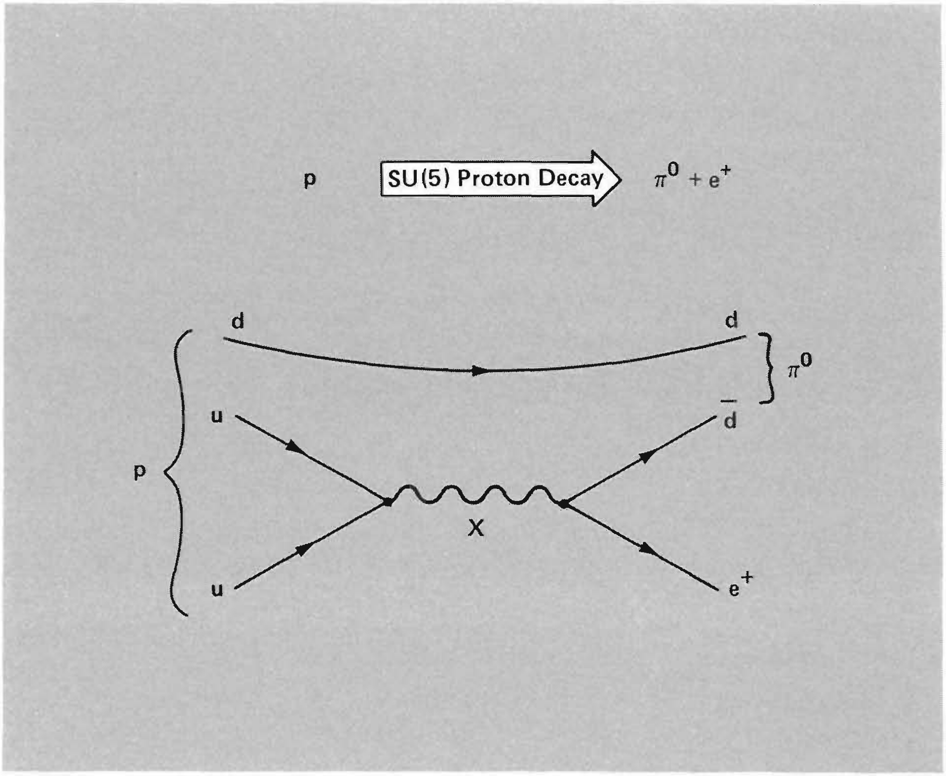


Fig. 6. The decay mode of the proton predicted by the minimal unification symmetry SU(5). The expected decay products are a neutral pion π^0 and a positron e^+.

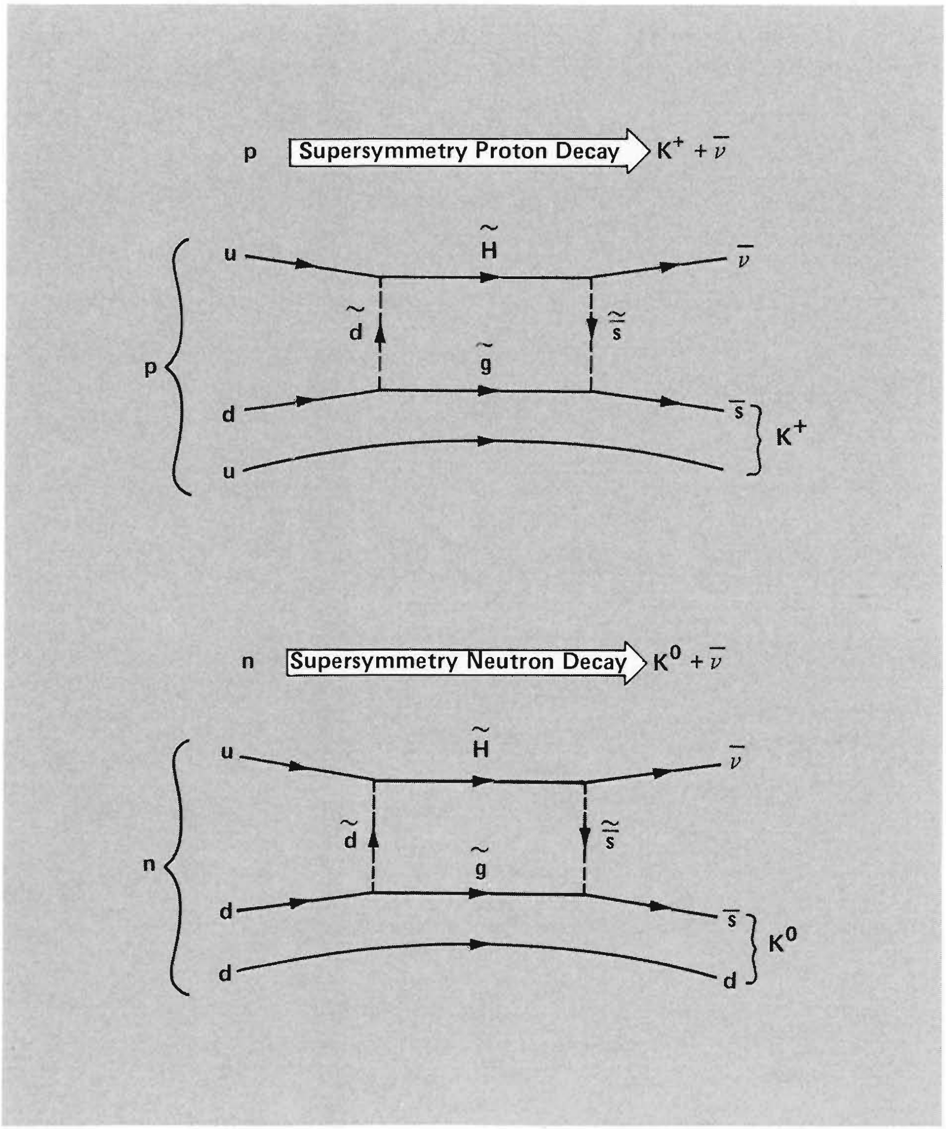


Fig. 7. The dominant proton-decay and neutron-decay modes predicted by supersymmetry. The expected decay products are* K *mesons (*K^+ *and* K^0*) and neutrinos (*$\bar{\nu}$*).

pected rate, then minimal SU(5) is in serious trouble.

The dominant decay modes predicted by minimal SU(5) for the nucleons are

$$p \rightarrow \pi^0 e^+$$

and

$$n \rightarrow \pi^- e^+ . \qquad (9)$$

These processes involve the exchange of a so-called X or Y boson with mass of order M_G (Fig. 6), so that the predicted proton lifetime τ_p is

$$\tau_p \sim \frac{M_G^4}{m_p^5} \frac{\hbar}{c^2} \sim 10^{28\pm2} \text{ years}, \qquad (10)$$

where m_p is the proton mass.

Recent experiments, especially sensitive to the decay modes of Eq. 9, have found $\tau_p \geq 10^{32}$ years, in contradiction with the prediction. Hence minimal SU(5) appears to be in trouble. There are, of course, ways to complicate minimal SU(5) so as to be consistent with the experimental values for both $\sin^2\theta_W$ and proton decay. Instead of considering such ad hoc changes, we will discuss the unexpected consequences of making minimal SU(5) globally supersymmetric. The parameter $\sin^2\theta_W$ does not change considerably, whereas M_G increases by an order of magnitude. Hence, the good prediction for $\sin^2\theta_W$ remains intact while the proton lifetime, via the gauge boson exchange process of Fig. 6, naturally increases and becomes unobservable.

It was quickly realized, however, that other processes in supersymmetric SU(5) give the dominant contribution towards proton decay (Fig. 7). The decay products resulting from these processes would consist of K mesons and neutrinos or muons, that is,

$$p \rightarrow K^+\bar{\nu}_\mu \text{ or } K^0\mu^+ , \qquad (11)$$

and so would differ from the *expected* decay products of π mesons and positrons. This is very exciting because detection of the products of Eq. 11 not only may signal nucleon decay but also may provide the first signal of supersymmetry in nature. Experiments now running have all seen candidate events of this type. These events are, however, consistent with background. It may take several more years before a signal rises up above the background.

Experiments. An encouraging feature of the theory is that low-energy supersymmetry can be verified in the next ten years, possibly as early as next year with experiments now in progress at the CERN proton-antiproton collider.

Experimenters at CERN recently discovered the $W^{\pm}$ and Z^0 bosons, mediators of the weak interactions, and produced many of these bosons in high-energy collisions between protons and antiprotons (each with momentum $\sim$ 270 GeV/c). For example, Fig. 8 shows the process for the generation of a W^- boson, which then decays to a high-energy electron (detectable) and a high-energy neutrino (not detectable). A single electron with the characteristic energy of about 42 GeV was a clear signature for this process.

However, also present in the CERN data were several different kinds of *anomalous* events (events that cannot be described by the standard model). Some of these have signatures characteristic of the predictions of supersymmetry.

For example, events were seen that contained one, two, or three hadronic jets and nothing more, which can be interpreted as a signal for either squark or gluino production (Figs. 5 and 9). A two- or four-jet signal is canonical, but these events can look like one- or three-jet events some fraction of the time.

Further, the so-called UA-1 Collaboration at CERN found six events with two jets, a high-energy electron, and some missing energy. This is the characteristic signature of top quark production via W decay (Fig. 10), and thus these events may be evidence for top quarks. But there is also an event predicted by supersymmetry with the same signature, namely, the production of about 40-GeV squarks (Fig. 11). It will take many more events to disentangle these two possiblities.

The CERN proton-antiproton collider began taking more data in September 1984 with momentum increased to 320 GeV/c per beam and with increased luminosity. If the supersymmetric partners exist at these energies, they may be discovered during the next year. If, however, such particles are not seen, then we must wait for the next generation of high-energy accelerators.

Hopefully, it will not be too long before we learn whether or not the underlying structure of the universe possesses this elegant, highly unifying type of symmetry. ■

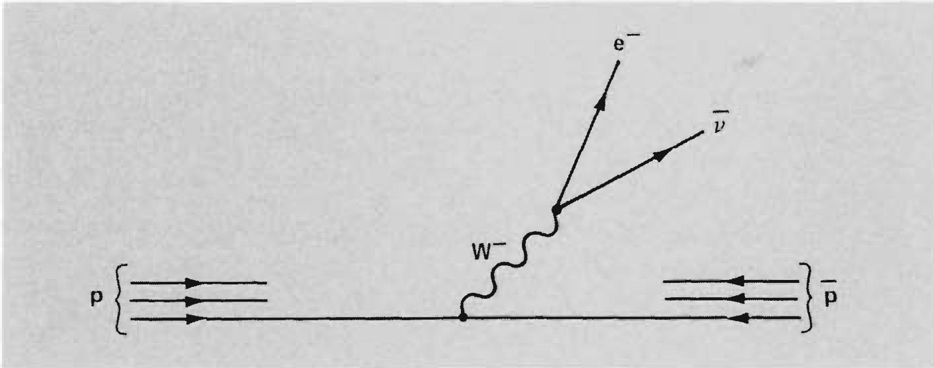


***Fig. 8. The generation, in a high-energy proton-antiproton collision, of a* W⁻ *particle, which then decays into an electron* (e⁻) *and an antineutrino* ($\bar{\nu}$).**

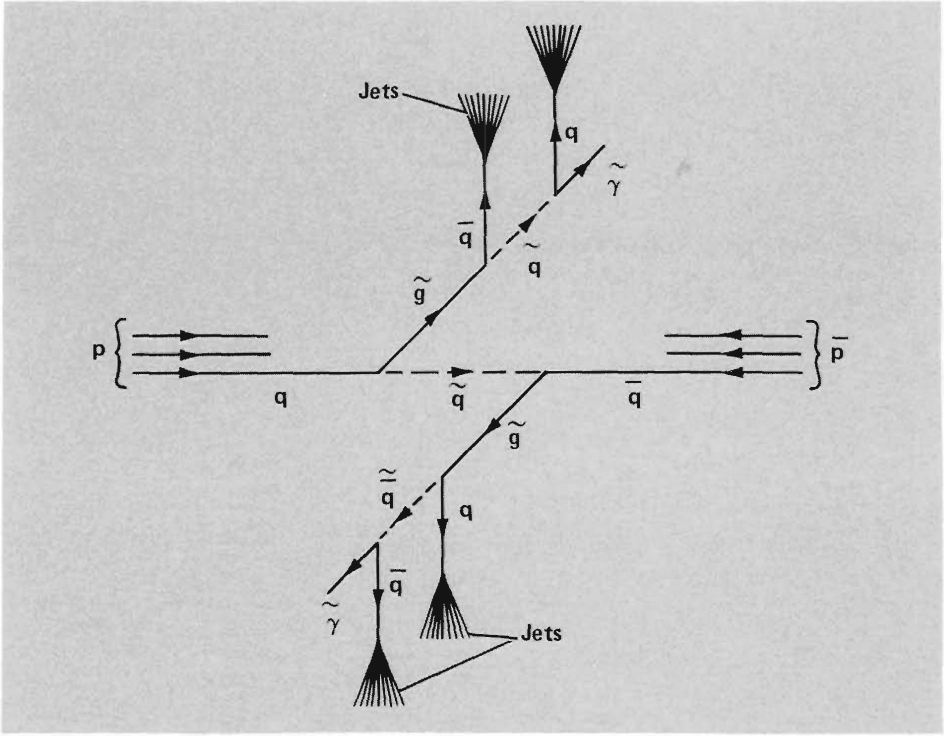


Fig. 9. A proton-antiproton collision involving supersymmetric particles (gluinos* $\tilde{g}$*, squarks* $\tilde{q}$*, antisquarks* $\tilde{\bar{q}}$*, and photinos* $\tilde{\gamma}$*) that generates four hadronic jets.

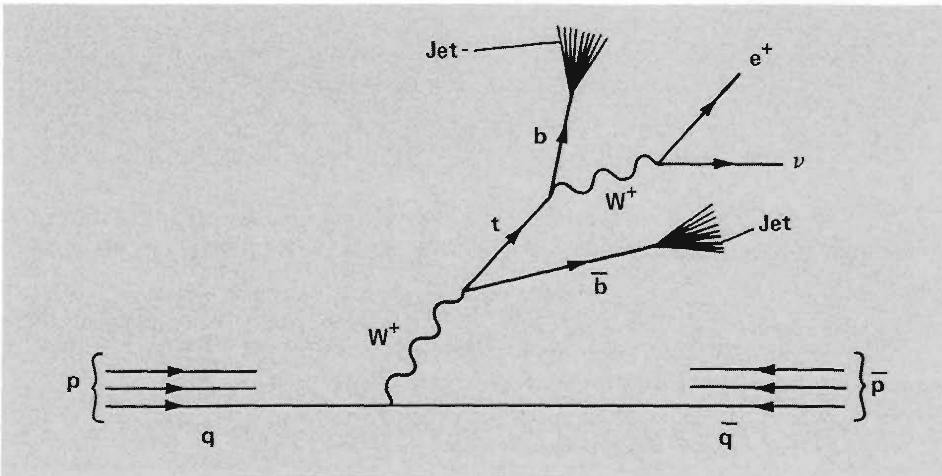


Fig. 10. Two-jet events observed by the UA-1 Collaboration at CERN can be interpreted, as shown here, as a process involving top quark* t *production.

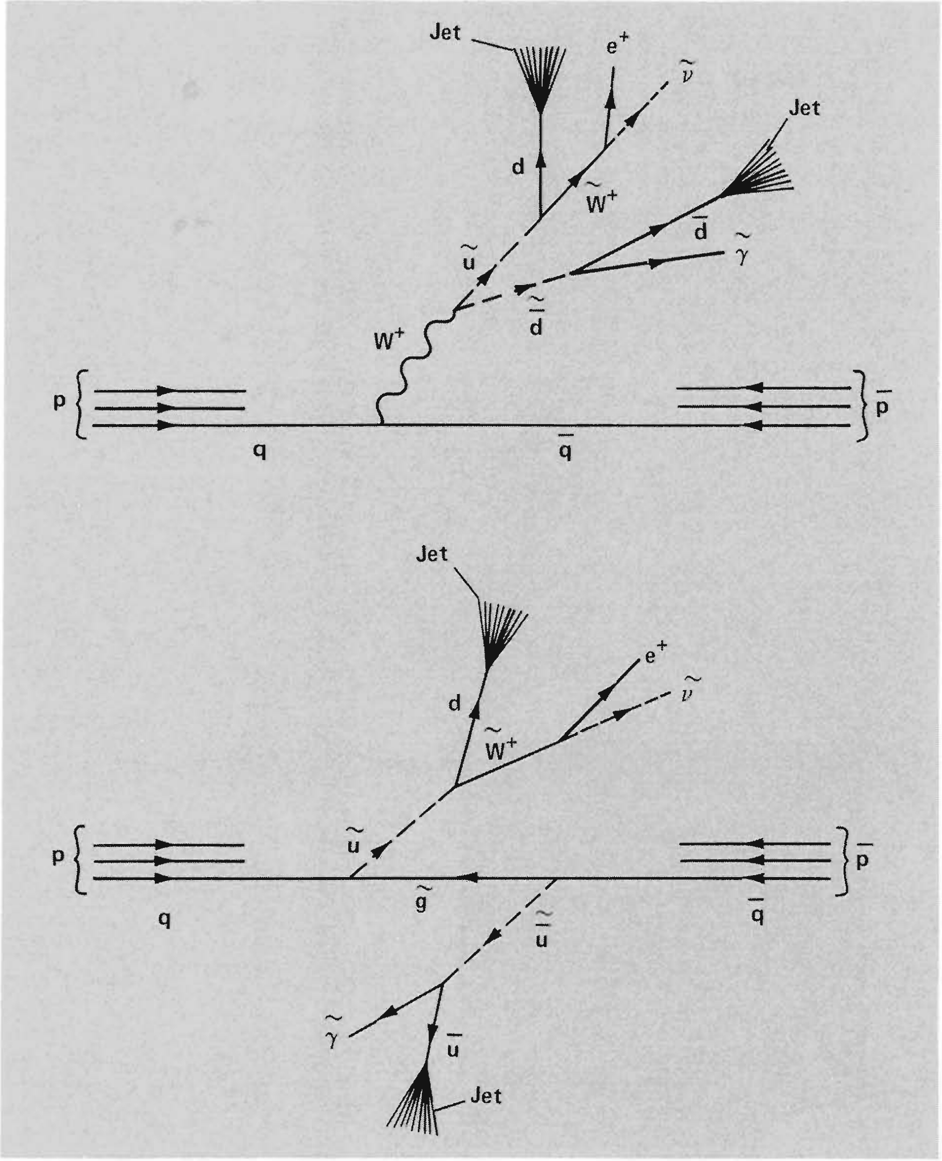
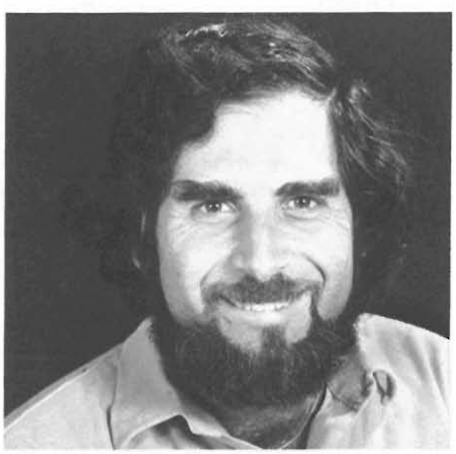


Fig. 11. The same event discussed in Fig. 10, only here interpreted as a supersymmetric process involving squarks and antisquarks.

Further Reading

Daniel Z. Freedman and Peter van Nieuwenhuizen, "Supergravity and the Unification of the Laws of Physics." *Scientific American* (February 1978):126-143.



Stuart A. Raby did his undergraduate work at the University of Rochester, receiving his B.Sc. in physics in 1969. Stuart spent six years in Israel as a student/teacher, receiving a M.Sc. in physics from Tel Aviv University in 1973 and a Ph.D. in physics from the same institution in 1976. Upon graduating, he took a Research Associate position at Cornell University. From 1978 to 1980, Stuart was Acting Assistant Professor of physics at Stanford University and then moved over to a three-year assignment as Research Associate at the Stanford Linear Accelerator Center. He came to the Laboratory as a Temporary Staff Member in 1981, cutting short his SLAC position, and became a Staff Member of the Elementary Particles and Field Theory Group of Theoretical Division in 1982. He has recently served as Visiting Associate Research Scientist for the University of Michigan. He and his wife Michele have two children, Eric and Liat.